Q-Factor Measurement of Nonlinear Superconducting Resonators

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Abstract

A novel method, which combined a multi-bandwidth measurement and an extrapolation procedure, is proposed for extracting the loaded Q-factor (Q_L) with improved accuracy from non-Lorentzian resonances of nonlinear superconducting resonators.

Index Items: microwave measurement, superconducting microwave devices, Q-factor, nonlinearity.

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1. Introduction

The nonlinear microwave surface impedance ($Z_S=R_S+jX_S$) of high temperature superconductors, i.e., its power dependence $Z_S(P)$, is of interest both for practical applications and for fundamental materials understanding. The study of $Z_S(P)$ often relies on the resonant techniques via measuring the Q-factor and the resonant frequency of a superconducting resonator as a function of microwave power P. There is a need for an accurate extraction of the Q-factors from the measured nonlinear resonance curves, which are generally non-Lorentzian as will be shown below, for quantifying $Z_S(P)$ and exploring its origins.

2. Method

In the case of linear response, the well-known frequency dependence of the transmission loss $T(f)=|S_{21}(f)|^2$ for a transmission mode resonator is [1]

$$T(f)=\frac{|T(f_0)|}{1+\left[2Q_L(f-f_0)/f_0\right]^2} \tag{1}$$

where $T(f_0)=4\beta_1\beta_2/(1+\beta_1+\beta_2)^2$, β_1 and β_2 are the coupling coefficients. The resonance curve of $T(f)$ has a well defined Lorentzian shape and the loaded Q-factor, Q_L, of the resonator can be easily obtained by measuring the 3-dB (half power) bandwidth, Δf_{3dB}, of the transmission curve and the resonant frequency f_0.

$$Q_L = f_0/\Delta f_{3dB}. \tag{2}$$

When the resonator undergoes a nonlinear response, however, the resonance curve diverges from the Lorentzian shape and becomes asymmetric. This effect is illustrated in Fig.1 where the resonance curves of a microstrip resonator made from double-sided superconducting $YBa_2Cu_3O_{7-\delta}$ (YBCO) thin film are

measured as the input power is increased in 5-dB steps from –18 to 12 dBm. Similar effects were also observed in the resonators made from NbN and Nb films [2]. At low input power, the resonance curves are symmetric about the resonant frequency and can be fitted well with the Lorentzian functions though fluctuations due to small signal-to-noise ratio are noticeable. As the input power increases, the resonance curve broadens gradually and becomes asymmetric and non-Lorentzian, with the peak resonant frequency shifting to lower frequency and the insertion loss increasing. Strictly speaking, when the resonance curve is clearly non-Lorentzian, Eqs. (1) and (2) become invalid and thus the traditional 3-dB bandwidth measurement of Q_L is no longer applicable. In the literature, however, the 3-dB bandwidth measurement has been widely used for investigating nonlinear responses of superconducting materials without justification.

In order to extract Q_L from the non-Lorentzian resonance curve, the origin of the non-Lorentzian resonance has to be examined. Firstly, due to the $T(f)$ response of the resonator, the microwave power (P) coupled into the microstrip varies at different frequencies around the resonance. Secondly, when P is high enough, the surface impedance of the superconducting microstrip shows a power dependent behavior, namely, $Z_S=Z_S(P)$. Thus for nonlinear response, Z_S is dependent on frequency. And this variation of R_S and X_S will cause the overall resistance R and inductance L of the resonant circuit to vary with frequency. As a result, the different points on the resonance curve correspond to different LCR resonant circuits and thus have different effective values of Q_L and f_0. By rewriting Eq.(1), Q_L can be generally calculated as

$$Q_L = \sqrt{1/\tau - 1}\, f_0 /(f_R - f_L) \tag{3}$$

where the relative power transmission ratio $\tau = T(f_R)/T(f_0) = T(f_L)/T(f_0)$ and f_R-f_L is the bandwidth measured at τ. If choosing the value of τ to be 0.5, f_R-f_L=Δf_{3dB} and Eq.(3) turns back to Eq.(2). From the analysis above, we know that the invalidity of Eq.(3) comes from the fact that $P(f_R)$ and $P(f_L)$ are different from $P(f_0)$. It is easily to be realized that if we choose a larger value of τ, smaller differences between them can be expected and Eq.(3) will give an better approximation for the $Q_L(f_0)$ value.

Shown in Fig.2 is a typical non-Lorentzian transmission curve where P indicates the resonant peak, A and A' are the 3-dB points used in the traditional Q_L measurement, BB', CC' and so on indicate other pairs of reference points with different τ values. From every pair of these points, an approximate $Q_L(f_0)$ value can be obtained by measuring the bandwidth of the reference points. The accuracy of the obtained $Q_L(f_0)$ will be improved when the τ value gets larger. Theoretically, when τ of the chosen reference points is approaching 1, Eq.(3) will give an accurate $Q_L(f_0)$ value. Following the error analysis similar to that in [3], however, the relative uncertainty in measuring Q_L, which is the same as that in measuring the bandwidth of reference points, reads

$$\left|\frac{\Delta Q_L}{Q_L}\right| = \left|\frac{\Delta BW}{BW}\right| = \frac{1}{2\tau(1-\tau)}\Delta\tau\,. \tag{4}$$

As τ is approaching 1, the error in τ, which is mainly caused by the inaccuracy of the amplitude reading of the instrument, will cause very large error in the resulting Q_L. Here we use an extrapolation method to overcome the problem. First we measure a set of Q_L values as a function of τ and then extrapolate them to a common intercept at τ =1. The resulting value is a reasonable approximation for $Q_L(f_0)$.

3. Results

Depicted in Fig.3 is the calculated Q_L versus the chosen τ value of the reference points for the curve shown in Fig.2. It is found that when the chosen value of τ gets large, the resulting Q_L value gets smaller. This agrees well with theoretical expectation for the resonator will have a larger effective R at the frequency point with larger τ value. The difference between the Q_L value from traditional 3-dB bandwidth measurement (Q_L^{3dB}) and that obtained from the method presented above (Q_L^{ex}) is very prominent. While the Q_L value from traditional 3-dB bandwidth measurement (Q_L^{3dB}) is about 3200, the value obtained from the method presented (Q_L^{ex}) is just about 2400. The relative difference (Q_L^{3dB} - Q_L^{ex})/ Q_L^{ex} is larger than 30%.

We have measured the Q_L^{3dB} and Q_L^{ex} for the resonance curves in Fig.1. The value of (Q_L^{3dB} - Q_L^{ex})/ Q_L^{ex} are plotted as a function of the input power in Fig. 4 and the absolute values of Q_L^{3dB} and Q_L^{ex} are shown in the insert. When the power is not large, the Q_L^{3dB} is just small different from Q_L^{ex} and gives quite a good approximation for the $Q_L(f_0)$. As the input power increases, the difference between them gets large and Q_L^{3dB} diverges noticeably from Q_L^{ex}. In this case, the application of the 3-dB measurement should be avoided or it will underestimate the power dependence of Z_S and may even cause a misunderstanding of the origin of the nonlinearities.

4. Conclusions

In conclusion, we have proposed a method to extract the Q_L value at the resonant peak of a non-Lorentzian resonance curve that is widely observed in the investigation of microwave nonlinear response of superconducting materials. It is a combination of a multi-bandwidth measurement and an

extrapolation procedure. By analyzing the nonlinear response of a YBCO microstrip resonator as an example, it has been shown that the method proposed in this letter can do Q_L extraction with dramatically improved accuracy compared with the traditional 3-dB bandwidth measurement which may cause large error in the Q_L extraction when nonlinear responses are involved.

References

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Figure Captions

Figure 1. YBCO microstrip resonator. Transmission loss versus frequency for input power levels ranging from –18dBm to 12dBm in 5dB increments.

Figure 2. The typical non-Lorentzian resonance curve.

Figure 3. The Q_{L} values measured as a function of τ of reference points for the resonance curve shown in figure 2.

Figure 4. ($Q_{\mathrm{L}}^{\mathrm{3dB}}$ - $Q_{\mathrm{L}}^{\mathrm{ex}}$)/ $Q_{\mathrm{L}}^{\mathrm{ex}}$ versus input power obtained from the resonance curves in figure 1. The corresponding $Q_{\mathrm{L}}^{\mathrm{3dB}}$ (circle) and $Q_{\mathrm{L}}^{\mathrm{ex}}$ (triangle) are shown in the insert.

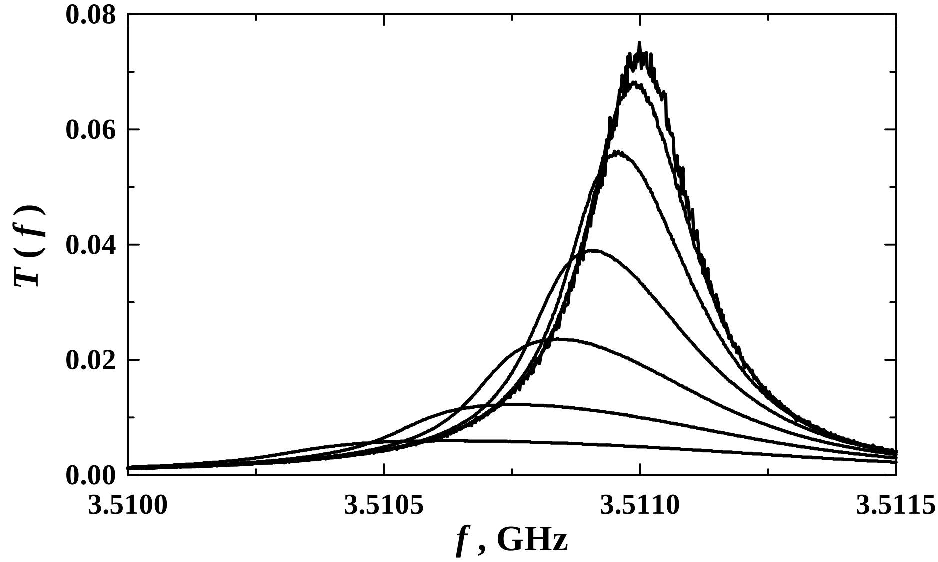

Figure 1 Rao *et al*.

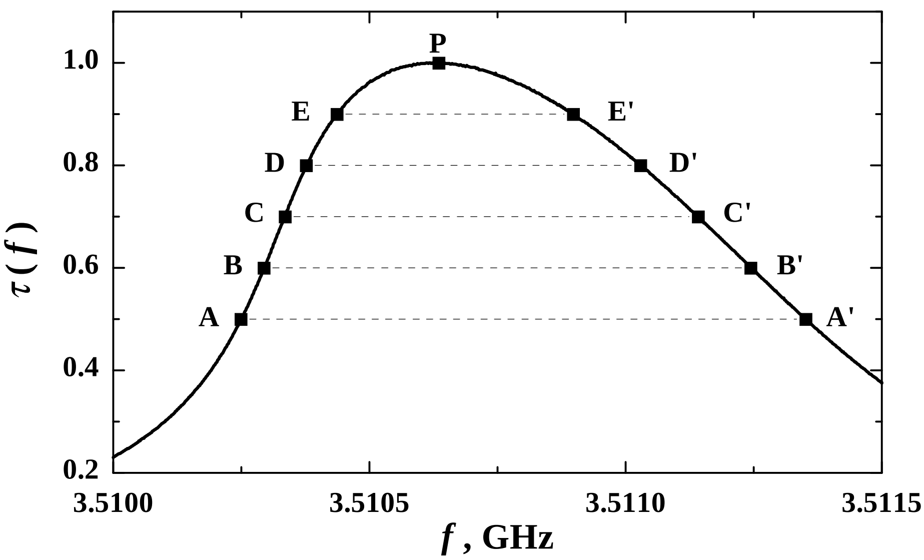

Figure 2 Rao *et al*.

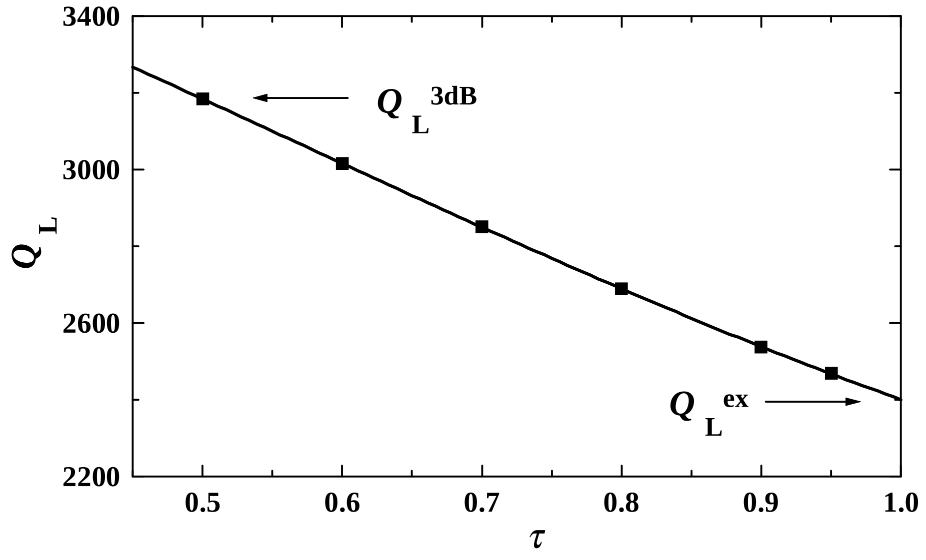

Figure 3 Rao *et al.*

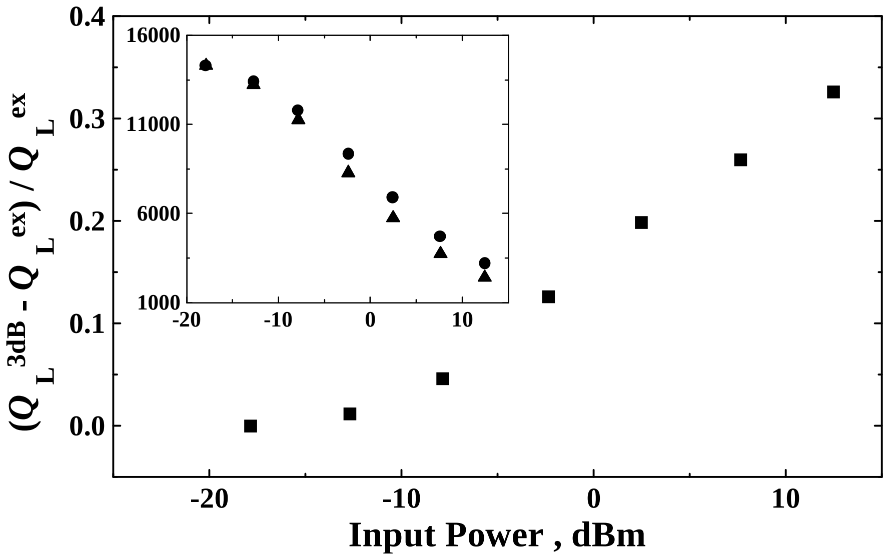

Figure 4 Rao *et al*.